Armed Services Technical Information Agency

Because of our limited supply, you are requested to return this copy WHEN IT HAS SERVED YOUR PURPOSE so that it may be made available to other requesters. Your cooperation will be appreciated.

AD 44999

NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY IMPLICATION OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, USE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

Reproduced by

DOCUMENT SERVICE CENTER

KNOTT BUILDING, DAYTON, 2, OHIO

UNCLASSIFIED

AD No.
ASTIA FILE COPY

NAVY DEPARTMENT
THE DAVID W. TAYLOR MODEL BASIN
WASHINGTON 7, D.C.

COMPARISON BETWEEN THEORETICALLY AND EXPERIMENTALLY DETERMINED NATURAL FREQUENCIES AND MODES OF VIBRATION OF SHIPS

by

R.T. McGoldrick

August 1954

Report 906

COMPARISON BETWEEN THEORETICALLY AND EXPERIMENTALLY DETERMINED NATURAL FREQUENCIES AND MODES OF VIBRATION OF SHIPS

by

R.T. McGoldrick

August 1954

Report 906
NS712-100

ABSTRACT

The results of vibration-generator tests and theoretical calculations of natural frequencies and normal modes of vibration on eight vessels of widely different types are discussed in this progress report. The tests were made with the TMB medium vibration generator producing driving forces up to 30,000 lb single amplitude. The calculations were made by means of the TMB electrical network analyzer. The errors in the calculated frequencies are tabulated. By using correction factors for the various modes based on the accumulated experimental data, more reliable estimates should be possible in the future. Data on the horizontal modes are also given, but coupling between horizontal and torsional motions requires further investigation before the reliability of predictions of horizontal modes can be forecast.

INTRODUCTION

The treatment of a ship together with the water moving with it as a free-free beam having numerous natural modes of flexural and torsional vibration covers an extensive literature which it is not intended to review in this report. The David Taylor Model Basin attempted to establish a set of equations applicable to the dynamics of this system in a report entitled "Recent Developments in the Theory of Ship Vibration."[1] This report is concerned chiefly with the experimental verification of the theory given therein.

It does not seem to have been sufficiently emphasized so far that the treatment of the hull and the surrounding water as a single vibrating mass is highly artificial. There are many limitations to this theoretical treatment among which are the uncertainties in evaluating the parameters representing bending rigidity, shearing rigidity, and effective mass which enter into the calculations themselves. As the frequency increases, a stage is reached at which all semblance to ordinary beam vibration disappears, and the equations can no longer be considered even approximately valid. Although this procedure has proved its utility in the calculation of natural frequencies and normal modes of vibration of hulls, it must be recognized that a beam vibrating on the surface of a dense fluid presents a dynamical system very different from that of a free-free beam in empty space.

The use of different values of virtual mass for different modes of vibration will increase the accuracy obtainable in the frequency calculations. A formula for estimating the variation in virtual mass for vertical modes has been proposed by E.H. Kennard,[2] but it is clear that the general dynamical problem cannot be treated by the equations given in Reference 1 if the mass must be considered to vary with the frequency.

---

[1]References are listed on page 16.

The chief aim of present ship vibration research is the prediction of the steady-state vibration of ships under the action of known sinusoidal exciting forces whether resonant or nonresonant. This report indicates the extent to which the theory has been experimentally verified up to the present time.

DESCRIPTION OF TMB EXPERIMENTAL METHODS

Although impact methods, such as the anchor-dropping method, may be used for the determination of the fundamental natural frequency or possibly the frequencies of the first two vertical modes, the systematic determination of the natural frequencies and normal modes of vibration of hulls requires in general the use of machines known as vibration generators. Such machines are built in a large variety of sizes and weights and may be either mechanical or electromagnetic. At present the most common type of hull exciter is the mechanical one having rotating eccentric masses.

In Reference 3 are described a variety of vibration machines, both portable and stationary, in use by the Taylor Model Basin, and in Reference 4 are summarized the results of a number of tests made on hulls by means of portable vibration generators. One such machine is illustrated in Figure 1. It is unnecessary to go into the details of the design of these machines here, but some discussion of the methods presently in use is pertinent especially since instrumentation for use in such tests is still undergoing development.

As shown in Reference 3, the mechanical vibration generator is usually adjusted to produce a unidirectional sinusoidal force but, if desired, certain machines may also be adjusted to produce a pure sinusoidal torque. For normal-mode and natural-frequency determinations the vibration generator is usually installed on the main deck either in the bow or in the stern. For determining the response to propeller forces it is installed in the stern, if possible, directly over the propellers.

Various procedures may be used in carrying out a vibration generator test. The particular one to be chosen depends not only on the type of recording equipment available but on the speed regulation of the vibration generator with the control system on hand. Two general procedures have proved satisfactory.

In the first the vibration generator is set for a given eccentricity and run at gradually increasing speeds until the maximum speed permissible with this eccentricity is reached. During this run the amplitude is recorded at some convenient point on the hull. A plot of amplitude against frequency gives the so-called resonance curve whose peaks indicate the natural frequencies of the hull. If the maximum speed reached with the initial eccentric setting is lower than the upper limit of frequencies to be explored on the vessel under tests, it is necessary to repeat the run with a lower eccentric setting which permits a higher machine speed. After the resonance curves have been plotted, the individual modes are explored by holding the speed constant at a value corresponding to each resonant frequency and measuring amplitudes along the hull with some form of portable amplitude-indicating meter.

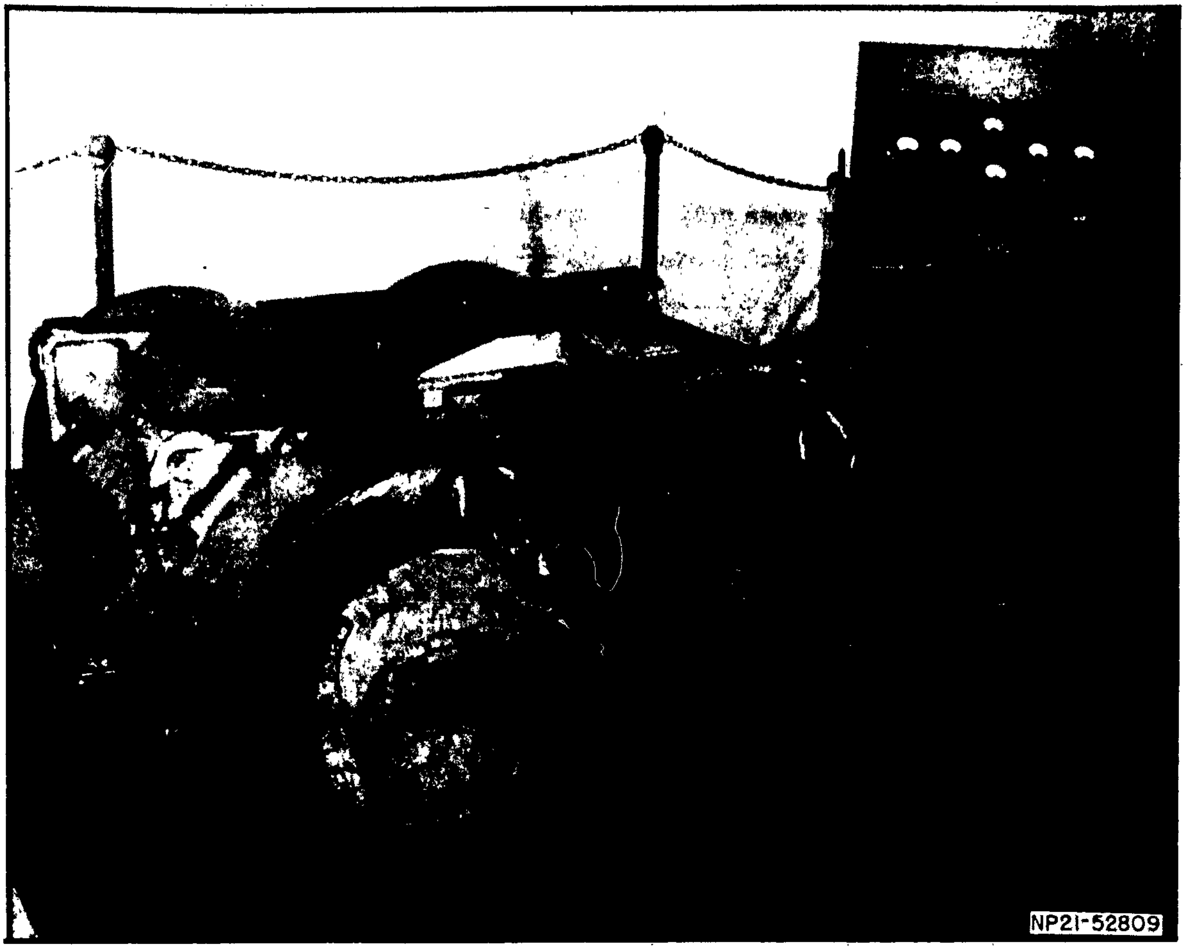


Figure 1 - TMB Medium Vibration Generator and Control Console

The other general method requires considerably more instrumentation but has compensating advantages. Amplitudes are measured simultaneously at numerous points along the hull by means of a recording system which consists of electrical vibration pickups connected through a multiconductor cable running the entire length of the ship to a recording system located in an instrument station supplied with amplifiers and multichannel oscillographs. Single excursions of the permissible speed ranges are made by the vibration generator at a rate of change of speed slow enough to develop the steady-state amplitudes during which time the recording system is in operation. Both the natural frequencies and the normal-mode patterns are then found by analysis of the oscillograms. In this case the control system should be capable of maintaining a uniform rate of change of speed.

At the time of writing this report the experience of the Model Basin, as far as full-scale hulls are concerned, has been limited to tests in which only a sinusoidal driving force has been produced. When the machine has been oriented to produce an athwartship driving force, it has usually been located on the main deck or at least on a deck sufficiently removed from the torsional axis that both torsional and horizontal flexural modes have been excited.

As horizontal and torsional motions of the hull may be coupled, the measuring technique must permit one to distinguish between these two motions. Such a technique requires measurement of phase as well as amplitude.

The TMB medium vibration generator can be adjusted to produce a pure couple by removing two eccentrics on the same side of the machine and replacing them after rotating them 180 deg. However, the couple thus produced is much less than the moment about the center of mass that is developed when the machine is adjusted for an athwartship sinusoidal force and installed on the main deck, since the lever arm is so large in the latter case. Up to the time of writing this report, experience in investigating the torsional modes of hulls by using the TMB medium vibration generator when set for a pure couple is lacking. However, in the experiments scheduled to be conducted on a dry cargo vessel of the Mariner class under the auspices of Panel S-6 of the Society of Naval Architects and Marine Engineers, it is planned to run such a test.

Since the vibration generator permits the determination of the amplitude produced by a known exciting force, it may also be used for the determination of the effective exciting forces due to propeller action. However, details of this phase of the ship vibration research program will not be discussed in this report.

Since at resonance the amplitude of the hull is limited only by damping, it is theoretically possible to establish damping values from the resonance curves obtained during vibration generator tests. As shown in this report the damping "constants" thus determined are not actually constant but vary with the frequency.

The anchor-dropping technique has proved extremely useful in finding the frequency of the fundamental vertical mode of hulls, although on occasion it may excite a higher mode. It consists simply in letting the anchor fall a few feet and suddenly arresting it. The resulting impulse usually sets the hull into vertical vibration. A continuous recording of this vibration will give not only the natural frequency but the logarithmic decrement from which a value of the damping constant may also be derived.

CALCULATED AND EXPERIMENTAL NATURAL FREQUENCIES AND DAMPING FACTORS

Although a number of other vibration-generator tests have been run on naval and merchant vessels, the test data assembled here are limited to those obtained on vessels tested in a sufficient depth of water to eliminate shallow-water and side effects. Test results on eight vessels of widely varying types are available for study at this time.

All these vessels were tested with the TMB medium vibration generator described in Reference 5; the tests were conducted by either of the methods previously described.

The principal data on these vessels are given in Table 1. Profiles and midship sections are shown in Figure 2.

TABLE 1

Principal Data on Vessels Discussed in This Report

Vessel	Type of Vessel	Length between Perpendiculars, L ft	Beam, B ft-in	Mean Draft Full Load ft-in	Displacement Full Load tons	Test Displacement tons	Mean Test Draft ft-in	Depth, D ft-in	Moment of Inertia of Midship Section, ft^4 Vertical	Moment of Inertia of Midship Section, ft^4 Horizontal	Shear Area, ft^2 Vertical	Shear Area, ft^2 Horizontal	L/D	B/D	L/B	Maximum Shaft rpm	Number of Blades per Propeller	Maximum Blade Frequency cpm
[illegible] NIAGARA	Transport	400	58-0	15-6	6 740	5 500	12-11	37-0	2 620	5 832	2.46	8.20	10.8	1.57	6.9	188	4	752
[illegible] CHAS. R. WARE	Destroyer	383	40-10	14-8	3 400	3 200	13-8	23-10	567	1 289	1.88	2.57	16.1	1.71	9.4	350	4	1400
[illegible] L. KULAS	Ore Carrier	580	60-0	19-8½	* 19 500	* 19 500	19-8½	32-0	2 490	9 008	4.20	8.99	18.1	1.875	9.67	110	4	440
[illegible] A. PAUL	Ore Carrier	520	54-0	21-4½	* 16 200	* 16 200	21-4½	31-0	1 825	3 631	4.32	4.63	16.8	1.74	9.62	123	4	492
[illegible] ERE [illegible] QUETTE 21	Car Ferry	348	56-0	14-7	* 5 900	* 5 900	14-7	21-6	2 482	6 900	3.90	4.63	16.2	2.6	6.21	110	4	440
[illegible] LD COLONY [illegible] RINER	Dry Cargo	528	76-0	31-6	21 000	16 400	24-0	44-6	7 821	16 000	4.41	16.13	11.9	1.71	6.95	105	4	420
[illegible] NORTHAMPTON	Cruiser	664	70-3	24-0	17 100	16 280	24-0	52-9	10 800	20 600	8.03	27.1	12.59	1.33	9.45	340	4	1360
USS STATEN ISLAND	Icebreaker	250	63-6	25-9	6 660	4 500	22 9	39-1½	2 890	8 480	8.20	8.46	6.39	1.62	3.93	110	3	330

*Short tons.

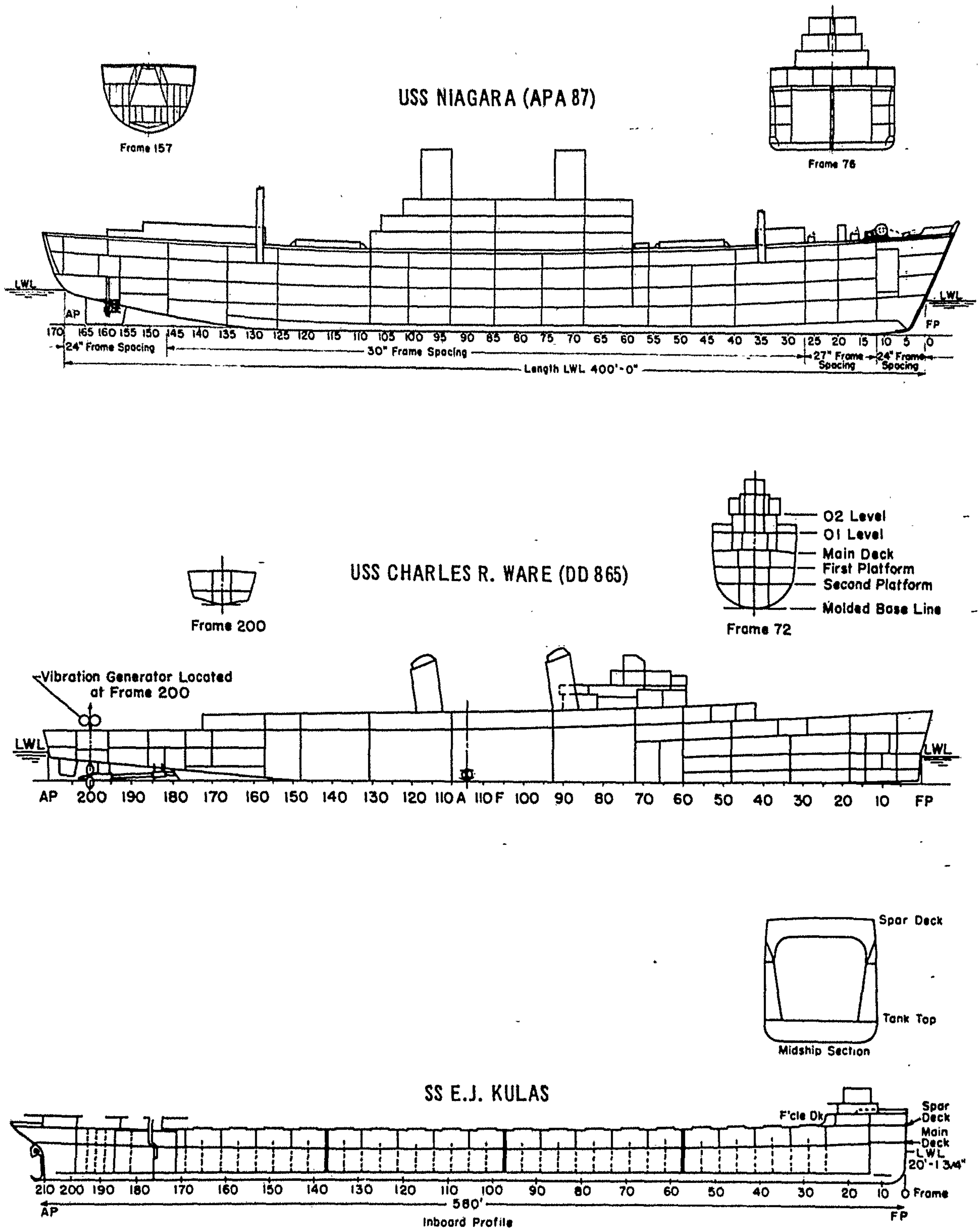
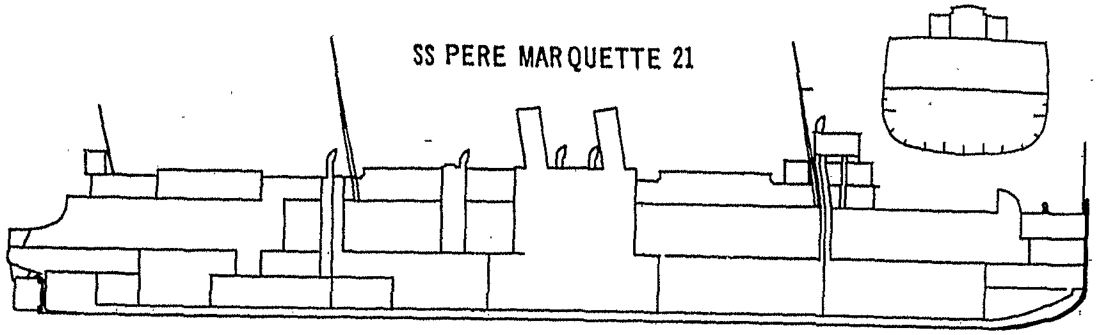


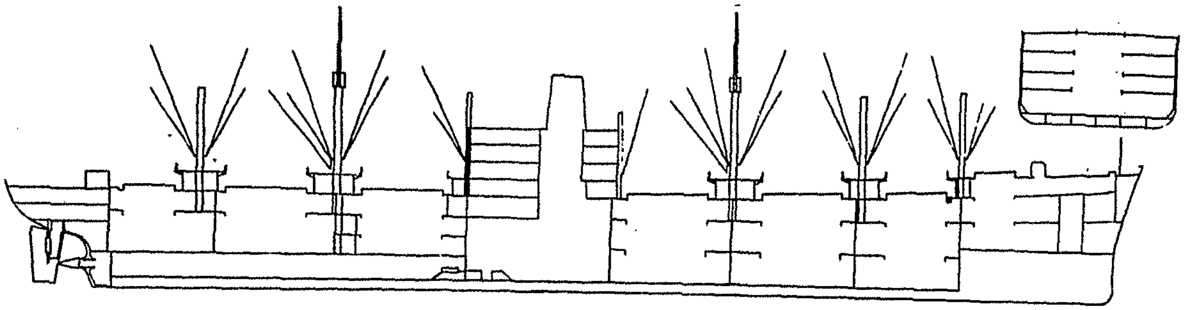
Figure 2 - Profiles and Sections of Vessels Tested

Profile of SS C.A. PAUL was not available.

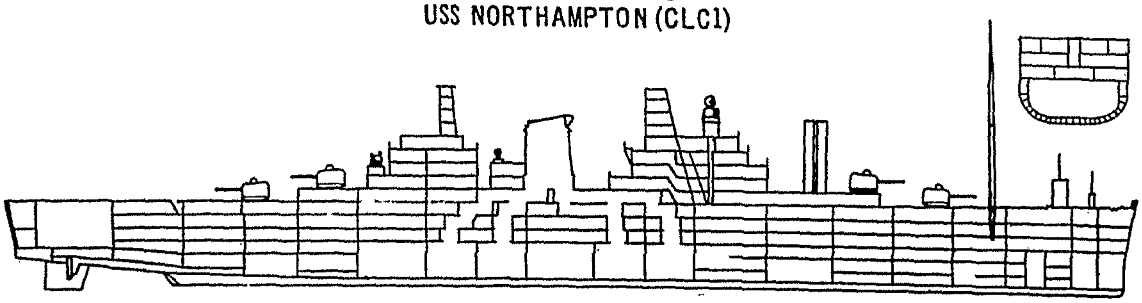
SS PERE MARQUETTE 21



SS OLD COLONY MARINER



USS NORTHAMPTON (CLC1)



SS STATEN ISLAND

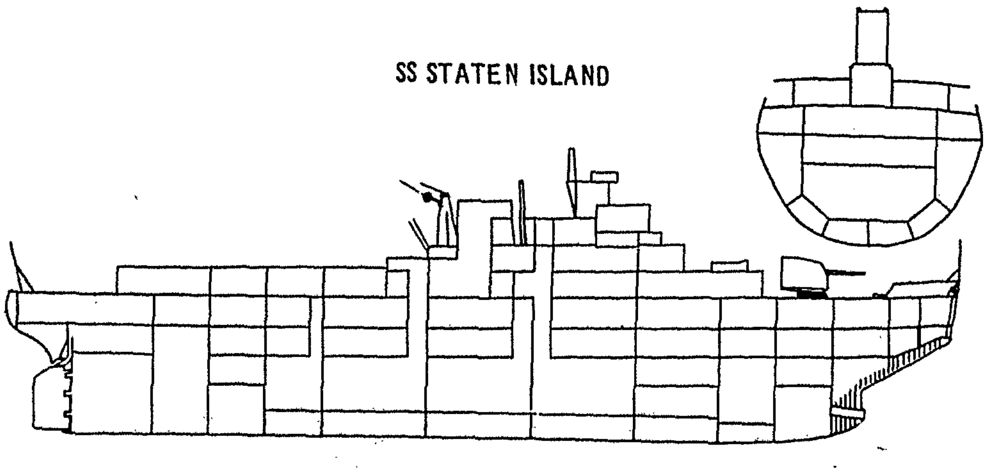


TABLE 2 - Comparison of Experimental and Calculated Frequencies of Vertical Modes

Vessel	Frequencies in cpm															
	1st Mode		2nd Mode		3rd Mode		4th Mode		5th Mode		6th Mode		7th Mode		8th Mode	
	exp	cal	exp	cal	exp	cal	exp	cal	exp	cal	exp	cal	exp	cal	exp	cal
NIAGARA	110	97	200	190	292	286	355	376	448	462						
CHARLES R. WARE	79	77	165	169	261	270	360	388								
E. J. KULAS		31	92	74	150	126	200	182	246	237	285	294	304	348	360	396
C. A. PAUL	45	38	106	96	168	167	210	241	312	315	354	390	432	459		
PERE MARQUETTE 21	112	113	224	249	346	385	512	524								
OLD COLONY MARINER	82	73	155	149	227	233	270	318								
NORTHAMPTON	68	64	133	130	204	207	288	283	359	357	437	434	500	500		
STATEN ISLAND	280	282	540	592	720	906										

TABLE 3 - Comparison of Experimental and Calculated Frequencies of Horizontal Modes

Vessel	Frequencies in cpm									
	1st Mode		2nd Mode		3rd Mode		4th Mode		5th Mode	
	exp	cal	exp	cal	exp	cal	exp	cal	exp	cal
NIAGARA	190	167	402	365	585	536				
CHARLES R. WARE	132	107	246	207						
E. J. KULAS		81	195*	189	320*	306	375*	444		558
C. A. PAUL			180	148	300	243				
PERE MARQUETTE 21	220	201	390	423						
OLD COLONY MARINER	118	107	280	234	350	369	435	495		
NORTHAMPTON	103	80	183	166	276	277	327†	405	392	530
STATEN ISLAND	420	375								

*Experimental determination of number of nodes not made; tabulation made to yield best agreement with calculated values.

†Uncertainty as to whether this is a flexural or a torsional mode.

TABLE 4 - One-Noded Torsional Modes Found by Vibration Generator Tests

Vessel	Frequency in cpm	Vessel	Frequency in cpm
NIAGARA	322	PERE MARQUETTE 21	–
CHARLES R. WARE	310	OLD COLONY MARINER	–
E. J. KULAS	262*	NORTHAMPTON	346*
C. A. PAUL	–	STATEN ISLAND	–

*Not positively identified as this mode.

The vertical modes are the most important as far as verification of present theory is concerned, and the results obtained for these modes will therefore be presented first. Table 2 gives a comparison between experimental and calculated frequencies of vertical modes.

As has been pointed out elsewhere, pure horizontal and pure torsional modes may not be found on certain vessels, and modes are possible in which both horizontal flexure and torsion occur simultaneously in various proportions. These are the so-called torsion-bending modes. The present stage of accumulation of experimental data is not sufficiently advanced to permit tabulation of definite torsion-bending modes in this report. Therefore only modes that have been identified either as horizontal flexural or torsional are tabulated. This does not imply, however, that the measuring technique employed was always adequate to establish that the mode was purely of one or the other type. Table 3 gives a comparison between calculated and experimental values of horizontal modes.

At this writing no reliable calculations are available relative to pure torsional modes for the vessels under consideration. There are given therefore in Table 4 only the scant experimental data on such modes.

Although the experimental determination of propeller exciting forces is not discussed here, some correlations between driving forces and amplitudes will be attempted and it should be noted that at resonance the amplitudes are determined by the damping as well as the driving force.

The theory of forced vibration given in Reference 1 makes use of a distributed viscous damping constant c which is the damping force per unit velocity per unit length and is also assumed to be proportional to the mass per unit length μ. Also defined in Reference 1 are the "effective mass" M_i and the "effective damping constant" C_i for each normal mode. It follows from these definitions that $C_i/M_i = c/\mu$ where the large letters represent the effective values for the ith normal mode and the small letters represent the values per unit length.

If the damping were actually of the type discussed in Reference 1, c would be independent of both amplitude and frequency, and it might be expected that the values of c/μ would fall within a reasonably narrow range for all classes of ships and for all modes of vibration. The existence of a universal value of this ratio could be extremely useful in estimating hull amplitudes for resonant conditions. Since in reality the damping appears to increase with frequency, it seems more likely that the ratio $c/\mu\,\omega$ would remain constant (ω being the circular frequency of the mode). In a vibration generator test the energy input per cycle at resonance W is given by the relation

$$W = \pi F_0 y_0$$

where F_0 is the amplitude of the driving force and y_0 is the displacement amplitude of the hull at the location of the vibration generator ("the driving point").

Under the assumptions stated in Reference 1, the energy absorbed per cycle is given by the equation

$$W = \int_0^L \pi\, c\, \omega\, y^2\, dx = \frac{\pi\, c\, \omega}{\mu} \int_0^L \mu\, y^2\, dx$$

since c/μ is assumed constant.

By equating these two energy expressions there is obtained the formula

$$\frac{c}{\mu} = \frac{F_0\, y_0}{\omega \int_0^L \mu\, y^2\, dx}$$

from which c/μ may be evaluated if the driving force, the amplitude at all points along the hull, and the mass per unit length including virtual mass are given.

In Table 5 are given values of c/μ and $c/\mu\,\omega$ obtained from data taken for vertical modes during vibration-generator tests on the vessels under discussion by the use of this equation. The average of all values of $c/\mu\,\omega$ given in the last column of Table 5 is 0.034.

An estimate of damping can also be made by observing the rate of decay of free vibration which yields the logarithmic decrement. If the damping were viscous and proportional to mass, the logarithmic decrement would be related to the ratio c/μ by the equation

$$\delta = \frac{\pi}{\omega}\, \frac{c}{\mu}$$

where ω is the natural circular frequency of the mode in question. It would also follow that the logarithmic decrement would vary inversely with the frequency of the mode.

To date, the few decrement observations reported were made during anchor-drop tests, a method that usually excites appreciably only the fundamental mode. Almost identical values of δ were found for the fundamental vertical mode on two destroyers of considerably different displacement. Of these the value for the CHARLES R. WARE was 0.022. As the frequency was 79 cpm, this corresponds to a c/μ of 0.058 and to a value of $c/\mu\omega$ of 0.007.

In deriving damping values for the horizontal modes it is to be noted that the value of μ will be different since the virtual mass values are different for the two cases. In Table 6 are given the damping factors derived for these modes. The average of all values of $c/\mu\omega$ given in the last column of Table 6 is 0.041.

TABLE 5

Damping Factors Derived from Vibration Generator Tests under Resonance Conditions (Vertical Modes Only)

Vessel	Mode	ω rad/sec	c/μ 1/sec	$c/\mu\omega$	Driving Force tons	Driving Point Single Amplitude ft
NIAGARA	1st	11.5	0.49	0.043	0.51	0.0011
	2nd	20.9	0.41	0.019	1.68	0.0021
	3rd	30.5	0.83	0.027	3.57	0.0014
	4th	37.1	2.6	0.067	5.27	0.0058
	5th	46.8	2.20	0.047	8.44	0.0049
CHARLES R. WARE	1st	8.2	0.17	0.021	0.30	0.0050
	2nd	12.2	0.17	0.010	1.32	0.0070
	3rd	27.3	0.31	0.014	3.32	0.0031
	4th	37.6	1.3	0.035	6.29	0.0092
E. J. KULAS	5th	29.8	0.80	0.027	2.77	0.0006
C. A. PAUL	1st	4.71	0.029	0.006	0.16	0.0079
	2nd	11.1	0.114	0.010	0.76	0.0064
PERE MARQUETTE 21	1st	11.7	0.168	0.014	0.89	0.0052
NORTHAMPTON	2nd	13.9	0.298	0.021	1.21	0.0008
	3rd	21.4	0.512	0.024	2.86	0.0007
	4th	30.2	0.722	0.024	5.71	0.0004
	5th	37.6	1.33	0.035	8.84	0.0004
	6th	45.8	2.55	0.056	12.95	0.0001
	7th	52.4	7.80	0.149	17.19	0.0002
STATEN ISLAND	1st	29.3	0.976	0.033	2.81	0.0038
				(avg)0.034		

TABLE 6

Damping Factors Derived from Vibration Generator Tests under Resonance Conditions (Horizontal Modes Only)

Vessel	Mode	ω rad/sec	c/μ 1/sec	$c/\mu\omega$ dimensionless	Driving Force tons	Driving Point Single Amplitude ft
NIAGARA	1st	19.9	0.293	0.015	1.76	0.0030
	2nd	42.1	0.943	0.022	6.79	0.0010
	3rd	61.3	2.49	0.041	5.98	0.0003
CHARLES R. WARE	1st	13.8	0.085	0.006	0.25	0.0044
NORTHAMPTON	1st	10.8	0.735	0.068	0.71	0.0008
	2nd	19.2	0.975	0.051	2.32	0.0007
	3rd	28.9	1.61	0.055	5.36	0.0004
	4th	34.2	2.15	0.063	7.14	0.0003
	5th	41.1	1.82	0.044	10.71	0.0002
STATEN ISLAND	1st	44.0	3.62	0.082	2.81	0.0007
				(avg) 0.041		

FORCED RESONANT VIBRATION ESTIMATES BASED ON EXPERIMENTAL DATA

The problem of estimating the forced vibration of a vessel in its design stage is one of considerable importance to the naval architect. Whereas much progress is to be expected in this field in the near future, some forecasting is possible even with the meager experimental data available at the present time.

It is assumed in this discussion that the vertical and horizontal components of the exciting force have already been predicted. It is also assumed that the natural frequencies and normal modes of vertical and horizontal vibration have been calculated by methods previously discussed. Torsional or torsion-bending modes are not considered at this time.

It is attempted here to deal only with the case of forced vibration at a hull resonance in which case the amplitude is limited only by the damping. The values of $c/\mu\omega$ given in Tables 5 and 6 should be helpful in making the forecast.

In accordance with the equations given on page 24 of Reference 1 the amplitude of resonant forced vibration will be given by the relation

$$y(x) = \frac{F\, X_i\,(x_0)}{\omega_i^2 \left[\frac{c}{\mu\omega_i}\right] \int_0^L \mu\, X_i^2\,(x)\; dx} \cdot X_i\,(x)$$

In order to use this relation the normal-mode shape must first have been determined by calculation for the mode in question, designated as the ith mode. The normal-mode shape is plotted in dimensionless units with the value unity at the after perpendicular giving the curve $X_i(x)$. The coefficient of $X_i(x)$ in the above equation will have the dimension of length. F is the single amplitude of the driving force in tons; $X_i(x_0)$ is the value of the normal-mode ordinate at the driving point; ω_i is the resonance circular frequency in radians per second; the value of $c/\mu\omega_i$ is the average of the values tabulated in Table 5 or Table 6. The value of $\int_0^L \mu X_i^2(x)\,dx$ is obtained by graphical integration, μ being the mass per unit length including virtual mass in ton-sec^2/ft^2. The numerical value of the coefficient of $X_i(x)$ will give the estimated single amplitude at any desired point along the hull in feet. It should be noted that the assumption is made that at resonance the components of modes other than the resonant mode are negligible.

COMPARISON BETWEEN THEORY AND EXPERIMENT

The vertical modes are unquestionably the best to consider for verification of the basic theory of ship vibration because of the symmetry with respect to the vertical plane through the fore-and-aft centerline. To show the degree of correlation between the calculated and actual frequencies of these modes (summarized in Table 2) the percentage errors in the calculated values are given in Table 7. Both the algebraic average error and the absolute average error are given at the bottom of this table. The former indicates the tendency of the method used to underestimate the frequencies of the first two modes, to give the frequency of

TABLE 7

Percentage Error in Calculated Frequencies of Vertical Modes

Vessel	Error in Percentage of Experimental Value							
	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode	6th Mode	7th Mode	8th Mode
NIAGARA	−12	− 5	− 2	+ 6	+3			
CHARLES R. WARE	− 3	+ 2	+ 3	+ 8				
E.J. KULAS		−20	−16	− 9	−4	+ 3	+12	+10
C.A. PAUL	−16	− 9	− 1	+15	+1	+10	+ 6	
PERE MARQUETTE 21	+ 1	+11	+11	+ 2				
OLD COLONY MARINER	−11	− 4	+ 3	+18				
NORTHAMPTON	− 6	− 2	+ 2	− 2	−1	− 1	0	
STATEN ISLAND	+ 1	+10	+26					
Average Algebraic Error	− 6.6	− 2.1	+ 3.3	+ 5.4	0	+ 4.0	+ 6.0	+10.0
Average Absolute Error	7.1	7.9	8.0	8.6	2.0	4.7	6.0	10.0

the third fairly accurately, and to overestimate the frequencies of modes beyond the third by gradually increasing amounts.

In the case of the NIAGARA it was thought at first that the underestimate of the frequency of the fundamental mode was due chiefly to the neglect of the contribution of the superstructure to the bending rigidity, and further calculations were made on the assumption that the superstructure was fully effective and also on the assumption that it was partially effective. The former assumption gave a value of frequency in perfect agreement with the experimental value. This appeared to indicate the importance of including the superstructure in the moment of inertia; but the NIAGARA is the only one of the eight vessels under discussion which has any appreciable superstructure and errors of the same order as found on the NIAGARA with the superstructure neglected have been found on vessels with no appreciable superstructure.

Recent comparisons between calculations on the analog and those on a digital computer indicate that part of the error found in the frequency of the fundamental mode may be ascribable to the analog itself.

As pointed out in Reference 1, the chief factors affecting the fundamental vertical mode are the bending stiffness EI and the virtual mass. The chief factors affecting the higher modes appear to be the shear stiffness factor KAG and the virtual mass. The virtual mass is believed to decrease in the higher modes. A lower virtual mass, however, would give higher frequencies and the calculated values for the higher modes are already too high. Moreover, the use of a virtual mass factor varying with frequency does not appear practical at this time.

By using lower virtual masses and lower shear stiffnesses it would no doubt be possible to show over-all improvement in the calculated values. It would be equally effective, however, to continue making the calculations by the present procedure and to apply to the values thus obtained correction factors equal to the algebraic average errors shown in Table 7. If this were done in the case of the NIAGARA, the frequencies of the first five modes would be predicted within 5 percent. A study is being initiated to determine the extent of systematic errors in the network analyzer itself.

Table 8 gives the percentage errors in the calculation of frequencies of the horizontal modes.

Although the data on horizontal modes are much scantier than those on vertical modes, the same trend in the errors is shown in the two cases and the same procedure may be adopted for the present as recommended for vertical modes, namely to carry out the calculation as outlined in Reference 1, and to apply the experimentally determined correction factors given in Table 8.

Because of the greater bending stiffness of the ship with respect to horizontal modes and the lower virtual-mass effect of the surrounding water, the frequency of the fundamental horizontal mode is usually considerably higher than the frequency of the fundamental vertical mode. It has been found more difficult to excite as many clearly defined horizontal modes as

TABLE 8

Percentage Error in Calculated Frequencies of Horizontal Modes

Vessel	Error in Percentage of Experimental Value				
	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
NIAGARA	−12	− 9	−9		
CHARLES R. WARE	−19	−16			
E. J. KULAS	–	− 3	−5	+19	
C. A. PAUL	–	−18	−19.0		
PERE MARQUETTE 21	− 9	+ 9			
OLD COLONY MARINER	− 9	−17	+ 5	+14	
NORTHAMPTON	−22	− 9	0	+24	+35
STATEN ISLAND	−11				
Average Algebraic Error	−13.7	− 9.0	− 5.6	+19.0	+35
Average Absolute Error	13.7	11.6	7.6	19.0	35

vertical modes and at the present time the upper limit appears to be the fifth mode for vessels of usual length/beam ratios.

As has been pointed out in Reference 1, the horizontal flexural and torsional deformations of the hull may be combined into so-called coupled torsion-bending modes. Until further investigation of such modes has been made, it seems advisable to defer consideration of means of improving the accuracy of calculation of horizontal modes. However, it may be helpful to observe the trend of the errors in the calculations that have so far been made for modes that were considered horizontal, that is, by ignoring any torsional effects as given in Table 8.

CONCLUSIONS

As this is a progress report only tentative conclusions are offered at this time. The experimental evidence accumulated to date appears to substantiate the basic soundness of treating the ship as a free-free beam having bending and shearing flexibility, at least as far as the vertical modes of vibration are concerned. It is believed that a vibration analysis by the methods discussed in this report and in Reference 1 should be made for any new class of vessel even though the accuracy may be insufficient for selecting shaft speeds and number of blades per propeller with assurance that a critical condition can be avoided. The analysis will at least indicate the possibility of resonant conditions and in case the calculations indicate that vibration troubles may be encountered it may be advisable to provide substitute propellers or at least to have them designed. Systematic errors in the method of calculation can be corrected by applying factors based on average errors as shown in this report. This process should become more reliable as further experimental data are accumulated.

On the basis of experimentally determined damping values, forced vibration estimates are possible for resonant conditions provided the exciting forces have previously been predicted.

Much less accuracy appears attainable in the case of horizontal modes than in the case of vertical modes and the effect of coupling between horizontal and torsional motions requires further investigation.

ACKNOWLEDGMENTS

The information given in this report was derived from experimental and theoretical work involving most of the personnel of the Vibrations Division. In particular the author is indebted to Mr. N.H. Jasper, Mr. E. Kapiloff, Mr. J.T. Birmingham, Mr. R. Milam, and Mrs. A.W. Mathewson. The comments of Dr. E.H. Kennard in reviewing this report have also been helpful.

REFERENCES

1. McGoldrick, R.T., et al, "Recent Developments in the Theory of Ship Vibration," Taylor Model Basin Report 739 (Feb 1951).

2. McGoldrick, R.T., "Determination of the Hull Critical Frequencies on the Ore Carrier SS E.J. KULAS by Means of a Vibration Generator," Taylor Model Basin Report 762 (Jun 1951).

3. Robinson, Q.R., "Vibration Machines at the David W. Taylor Model Basin," Taylor Model Basin Report 821 (Jul 1952).

4. Hardy, V.S., "Vibration Studies of Ship Hulls by Means of Vibration Generators," Taylor Model Basin Report C-80 (Nov 1949).

5. Berdahl, E.O., "Construction and Operation of the Taylor Model Basin 5000-Pound Vibration Generator," Taylor Model Basin Report 524 (Apr 1944).

BIBLIOGRAPHY

Lewis, F.M., "Propeller Vibration," Society of Naval Architects and Marine Engineers Transactions (1935).

Baier, L.A. and Ormondroyd, J., "Vibration at the Stern of Single Screw Vessels," Paper presented at the spring meeting of the Society of Naval Architects and Marine Engineers, New Orleans (May 1952).

Budiansky, B. and Kruszewski, E.T., "Transverse Vibrations of Hollow Thin-Walled Cylindrical Beams," National Advisory Committee for Aeronautics Technical Note 2682 (Apr 1952).

Adams, E.J., "The Steady-State Response of a Ship Hull to a Simple Harmonic Driving Force Computed by a Digital Process," Taylor Model Basin Report 715 (May 1950).

Schade, H.A., "The Effective Breadth Concept in Ship-Structure Design," Paper No. 4 presented at annual meeting of Society of Naval Architects and Marine Engineers (1953).

Kapiloff, E., "Calculation of Normal Modes and Natural Frequencies of Ship Hulls by Means of the Electrical Analog," Taylor Model Basin Report 742 (Jul 1954).

INITIAL DISTRIBUTION

Copies	
12	Chief, BuShips, Library (Code 362) 5 Technical Library 1 Tech Asst to Chief (Code 106) 1 Applied Science (Code 370) 2 Noise, Shock, and Vibration (Code 371) 1 Preliminary Design (Code 420) 1 Underwater Explosion Res (Code 423) 1 Hull Design (Code 440)
2	Chief, Nav Res, Phys Sci Div, Math
1	Chief, BuOrd
2	DIR, USNRL, Shock and Vibration Section
1	CDR, USNOL
2	CDR, New York Naval Shipyard, Materials Lab, Vibration Section
2	CDR, Puget Sound Naval Shipyard, Vibration Section
2	CDR, Philadelphia Naval Shipyard, Scientific Section
2	CDR, San Francisco Naval Shipyard, Vibration Section
1	Administrator, U.S. Maritime Admin, Wash, D.C.
1	COMDT, U.S. Coast Guard, Washington, D.C.
1	DIR, Natl BuStand, Natl Applied Math Lab
1	SUPSHIP-INSORD, Newport News Shipbldg and Dry Dock Co., Newport News, Va.
1	Newport News Shipbldg and Dry Dock Co., Engin Tech Dept, Newport News, Va.
1	Sun Shipbldg and Dry Dock Co., Chester, Pa.
1	Prof. J. Ormondroyd, Univ of Michigan, Ann Arbor, Mich.
9	BJSM (Navy Staff)
3	CJS

NAVY-DPPO PRNC, WASH., D.C.